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Climatological Studies of the Large-Scale Circulation in the Northern Hemisphere

I. ZONAL AND MERIDIONAL INDICES AT THE 700-MILLIBAR LEVEL

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ABSTRACT—Within the framework of a detailed dynamic climatology of the large-scale circulation over the Northern Hemisphere, data from 15 yr of daily 700-mb maps have been used to investigate the temporal behavior of hemispherically averaged geostrophic zonal and meridional indices. A number of such indices were derived for both fixed and variable latitude bands. Their mean annual pattern, interannual variability, and time history through the 15 yr of record have been established.

The absence of statistically significant correlations between zonal and meridional flow on a short-term time scale (days to weeks) is established, while high correlations are apparent on the annual scale. The 15-yr record also indicates the sensitivity of the meridional indices to changes in the analysis techniques used to derive the hemispheric gridpoint data from the raw radiosonde observations. Finally, the capability of attaining a high resolution in time (about 3 days on the average) because of the daily data record allows a much more detailed description of the annual change in the large-scale circulation character than was previously possible by means of monthly averages. Certain times of the year appear to be fairly quiet while, at other times, distinct breaks in the circulation patterns are evident. This is a first indication that hemispheric wide singularities indeed exist.

1. INTRODUCTION

One of the most frequently used parameters for characterizing the state of the general circulation of a particular hemisphere has been the zonal index, following the early definitions by Allen (1940) and Rossby (1941, 1947). This index is usually defined as the hemispherically averaged geostrophic zonal wind component between latitudes 35° and 55°, obtained from either pressure or height differences between these latitudes. In recent years, this index has been called the temperate zonal index, and two more indices, the polar and the subtropical, covering adjacent latitudes are employed; this, for example, is the practice of the National Weather Service. Willett (1948) introduced an additional measure of the circulation behavior, the meridional index, defined as the absolute average value of the north–south component of the geostrophic flow, calculated for a specific latitude—in his case for 30°, 45°, and 60°N. He also calculated the correlation between simultaneous values of the zonal and meridional indices, using 52 overlapping 5-day mean periods. He concluded that: "this correlation is consistently, but not very significantly, negative both at sea-level (average value −0.30) and at the 3 km level (average value −0.18, one of seven coefficients positive)."

Further studies of these indices were continued, both to clarify their behavior in space and time and to relate their variations to a number of other quantities, especially in the quest for improved extended period forecasting. Riehl et al. (1950) established the existence of the annual variation of the zonal index with maximum in winter and minimum in summer. They also found that the region of maximal zonal flow (at sea level) migrates southward to near latitude 40°N with the approach of winter. In contrast, the meridional flow shifts only slightly north or south of 50°N with the seasons, and its maximum value varies less in magnitude than the zonal flow, albeit with the same character, namely, a maximum value in winter.

Along with the establishment of the seasonal variation of the zonal and meridional indices, the slightly negative correlation between the two was more or less generally accepted. The Glossary of Meteorology (Huschke 1957) defines low index as a state characterized by weak westerlies usually associated with stronger meridional flow and Haltiner and Martin (1957, p. 150) state that "High index represents a state of the general circulation with strong westerlies in midlatitudes and weak meridional flow, whereas low index has weak westerlies but strong meridional flow in these same latitudes."

Since there is no question that, on an annual basis, both zonal and meridional indices vary in parallel (i.e., both are large in winter and considerably smaller in summer, which means a positive correlation between the two), and since the above cited statements imply a negative correlation for a short period (daily or 5-day), then there should be some time interval over which the two indices are essentially uncorrelated. Obtaining indices averaged over such an interval (of the order of possibly weeks or months) would allow the use of both in regression equations with the assurance that they constitute statistically independent predictors, an extremely valuable property in developing statistical extended range forecasting methods.

Since we had available, for research into the large-scale dynamic climatology of the 700-mb level, a complete record of daily 700-mb data for the Northern Hemisphere covering a total of more than 15 yr, we decided to look into the specific behavior of these indices. In the first part of this paper, some results are compiled, related to the commonly used definition of the zonal and meridional indices, that force us to revise some of the earlier statements.[1] In the second part, a more detailed representation of the zonal and meridional flow character and related quantities for the "average year" is given. This representation, based on overlapping 10° latitude zones, allows us to view the overall climatic behavior without the restriction of using an arbitrarily fixed latitude band.

2. DATA AND COMPUTATIONAL PROCEDURES

From the Extended Forecast Division of the National Weather Service, we obtained a complete file of twice-daily, 700-mb height values for the Northern Hemisphere in the form of latitude-longitude gridpoint data for a 10°×10° diamond grid extending northward from 20°N to the pole. Thirty-six values were recorded at each latitude; for latitudes ending with 0 (20°, 30°, etc.), the gridpoints are at longitudes also ending with 0 (0°, 10°W, 20°W, etc., to 10°E), while for latitudes ending in 5 (25°, 35°, etc.), longitudes were given for 5°W, 15°W, and so forth. No values were given for 85°N, but a single value for the pole was included so that the total number of data points per map is 469.

The original data (stored on magnetic tapes) contained two 700-mb maps per day (0000 and 1200 GMT). On the basis of completeness of coverage, we selected the data representing the 1200 GMT map for our studies. The 15-yr period chosen for our work extends from Jan. 1, 1951, through Dec. 31, 1965. More data were available, but we decided to limit the data to an integral number of full years to avoid complications. Height values of the 700-mb level were recorded in units of feet with an accuracy of 5 ft.

A mathematical representation of the height field of each daily 700-mb map was prepared for this study by means of Fourier-Bessel coefficients (Kutzbach and Wahl 1965). Two sets of calculations were made: (1) a computation of the Fourier coefficients for the 36 height values along each latitude circle (20°, 30°, etc., or 25°, 35°, etc.) and (2) a representation of these Fourier coefficients by means of Bessel functions in the south–north direction. Thus, the Fourier coefficients A_n and B_n were computed for each day of the 15 yr. A complete set of these coefficients was available for all harmonics from $n=0$ (equivalent to the mean of the 36 values) to $n=18$. We found in all cases tested, however, that, in each set of 36 values, the first 12 harmonics explained over 97 percent of the variance. Thus, the remaining harmonics, 13–18, could be neglected in further utilization of this material; they represent the "noise" in the data, due primarily to truncation resulting from the 5-ft accuracy limitation of the gridpoint data.

The determination of the daily zonal and meridional indices for the Northern Hemisphere from the harmonic coefficients then becomes quite simple. The zonal index, Z, in meters per second, may be expressed by

$$Z=\Delta A_0 K(\phi) \tag{1}$$

where $\Delta A_0=A_0(55°)-A_0(35°)$ (i.e., the differences of the Fourier coefficients for $n=0$ at 55° and 35°N, which represent the mean height of the 700-mb level at the respective latitudes). The factor

$$K(\phi)=-\frac{g}{f(\phi)}\frac{1}{c} \tag{2}$$

introduces the geostrophic assumption where $f(\phi)$ is the Coriolis parameter (at 45°N), g is the gravitational acceleration, and $c=7.29\times10^6$ ft, the distance between 35° and 55°N. This latter factor converts the results to the metric system.

The meridional index in previous studies had been obtained from the average value (along one latitude) of the absolute height difference between adjacent points of the grid; that is, at distances of 10° longitude. We obtained the equivalent (and essentially identical) quantity by computing it from the square root of the absolute value of the north–south transfer of kinetic energy in this level across the particular latitude. The index is obtained by using the amplitudes of the Fourier components (for the nth harmonic) $C_n=(A_n^2+B_n^2)^{1/2}$ and calculating, for latitude, ϕ, the value of

$$M(\phi)=D(\phi)\left(\sum_{n=1}^{k} n^2C_n^2\right)^{1/2} \tag{3}$$

with

$$D(\phi)=\frac{3.048}{\sqrt{2}}\,\frac{g}{f(\phi)r(\phi)\cos\phi}.$$

This factor takes care of both the conversion of the amplitudes from feet to the metric system and the convergence of the meridians toward the pole.

To obtain an average value of the meridional index for the same latitudinal band as in Z, we averaged the five $M(\phi)$ for the latitudes 35°, 40°, 45°, 50°, and 55°N resulting in the meridional index, M. Z and M were computed for each day from Jan. 1, 1951, through Dec. 31, 1965. From this basic population, the behavior of the indices and their interrelationship on both a short- and long-term basis can be ascertained.

3. THE ANNUAL CYCLE IN THE INDICES

On the basis of the above calculations, one can plot the values of the two indices Z and M for any particular year. Figure 1 presents such a plot for the year 1957, randomly chosen from the available 15 yr of data. Actually, the values plotted are running 3-day averages of the index values. Averaging in this manner eliminates the spurious day-to-day variation caused by the differences in the map analysis but conserves the longer term variations. One can easily see in both curves a distinct annual cycle with higher values generally occurring in winter and lower values in summer. It is also evident, however, that the

[1] Many of these results were obtained by Riordan (1968) during completion of his M.S. thesis.

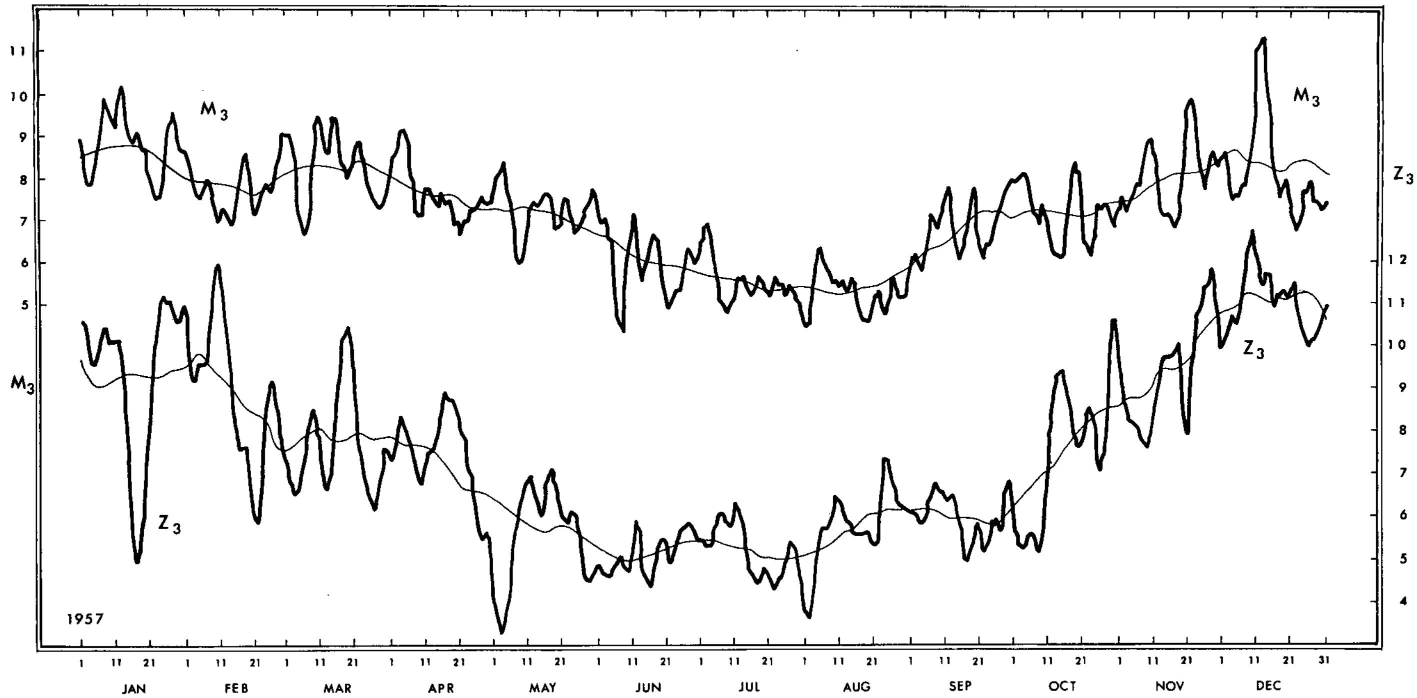

FIGURE 1.—Overlapping 3-day average meridional (M_3) and zonal (Z_3) hemispheric 700-mb indices (35°–55°N) for 1957. Units are m/s.

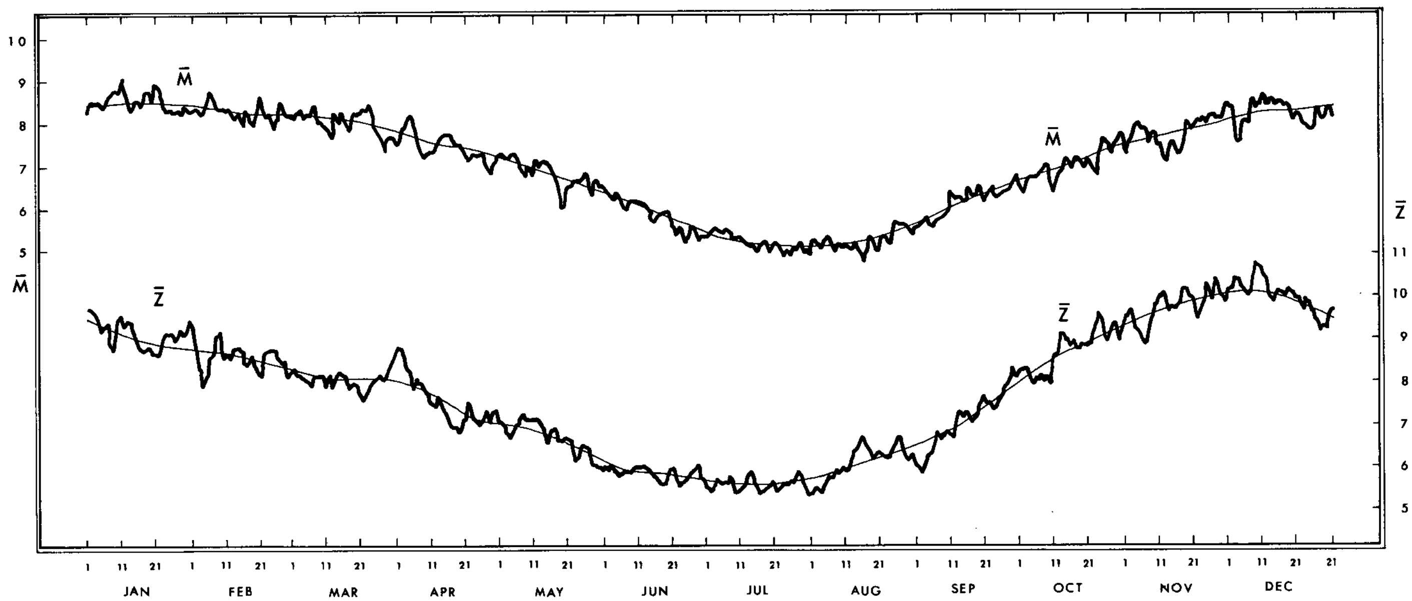

FIGURE 2.—Same as figure 1 for an average year, 1951–65.

short-term variations on a time scale of a few days or weeks are nearly of the same magnitude as the total variations between summer and winter. In other words, near-typical summer values can occur in midwinter while high values in summer may occasionally reach the typical winter level.

Note also that the magnitude of the short-term variations itself has a pronounced annual variation, especially in Z, while in M, these short-term variations are of a lesser magnitude and do not vary much from season to season. For example, the total range of average 3-day Z-values for January during the 15 yr of record was 10.57 m/s, from a maximum of 13.02 to a minimum of 2.45 m/s. In July, the total range is only 3.60 m/s, from a maximum value of 7.21 to a minimum of 3.61 m/s. For M, the total range in January was 4.6 (10.7–6.1) m/s, and for July it was 4.2 (7.5–3.3) m/s. It is apparent that both the absolute lowest and highest index values for Z occur in winter; the lowest daily value (Z=2.15 m/s) occurred on Jan. 7, 1956, and the highest (Z=14.44 m/s) on Dec. 12, 1965.

To clearly isolate the annual cycle, we computed daily 15-yr averages and smoothed them by running 31-day averages centered on the middle day of that "month." Figure 2 shows these two average annual index curves, using values for every 5th day of the average year. Clearly,

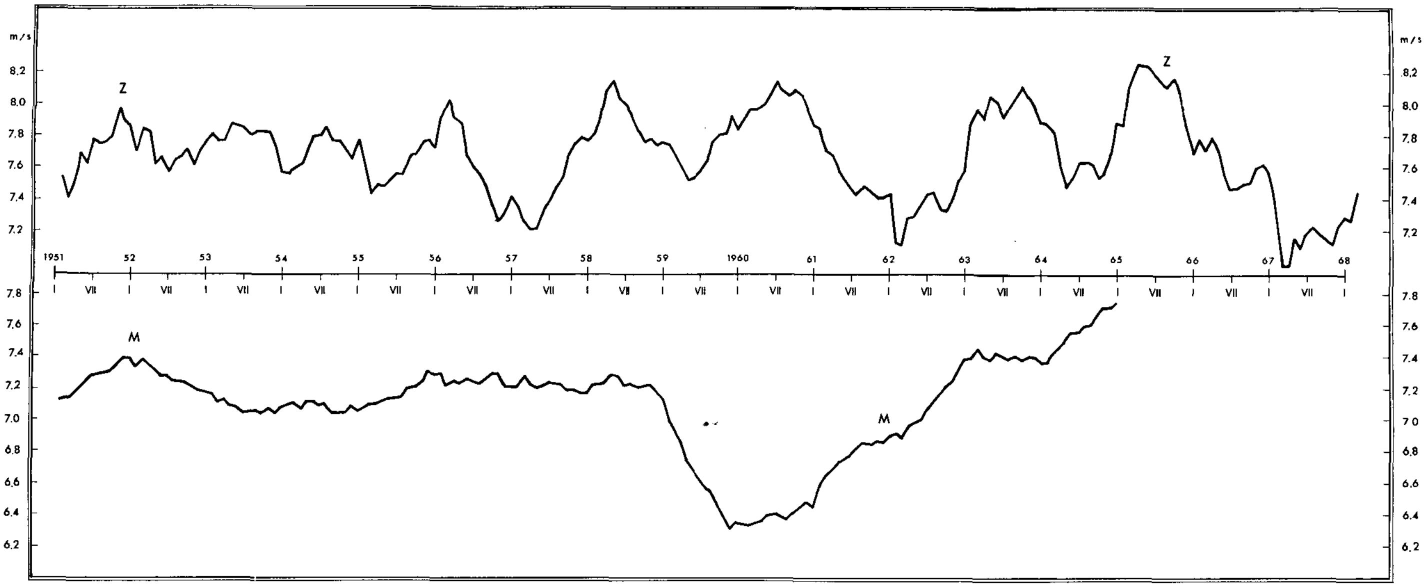

FIGURE 3.—Time history of zonal and meridional indices. Running 12-mo averages are plotted against the first month of "year" (e.g., the value plotted at 1951 I represents year 1951 I–XII, that at 1951 VII is year 1951 VII–1952 VI, etc.).

these two average curves are highly and positively correlated, with high values in winter and low ones in summer.

However, the actual time of maxima and minima is not the same in the two quantities. On these smoothed curves, the dates of maxima and minima for Z are December 4 and July 19, respectively. For M, the maximum occurs on January 13 and minimum on August 3. The Z-maximum thus occurs about 40 days before that of M while the Z-minimum occurs about 15 days earlier than the M-minimum. This suggests that the two quantities may correlate best at some lag, with Z preceding M; indeed, the best (positive) correlation is found at a lag of approximately 24 days. More will be said about this later. The occurrence of the highest Z-values in early December (i.e., before the winter solstice—an unusual behavior for an annual cycle in any weather element) is related to the manner in which the Z-index is defined. In midwinter, the maximum westerlies in the 700-mb level can occur at latitudes 40°, 35°, or even 30°N. Consequently, during January, February, and sometimes as early as the second half of December, the index interval as defined above (i.e., the latitude band 35°–55°N) may not even contain the actual maximum of the westerlies. Averaged over the 20° of latitude, one finds, for those times, somewhat lower values than in early December when the latitude of the maximum westerlies is still well north of 35°N. This same problem does not apply to the meridional index, M. In this element, the zone of maximum values is generally quite broad; the actual (long-term average) maximum is usually around latitude 50°–60°N, and an average index based on the usual 20° latitude belt is quite satisfactory. Clearly, the choice of the 35°–55°N index is not an optimal choice, especially for the zonal flow. If one uses this parameter, he should keep in mind the possibility of such problems as indicated above, especially during the winter months.

4. LONG-TERM VARIATIONS IN Z AND M

Eliminating the above-discussed annual cycle by means of 12-mo running averages, one can study the long-term behavior of these indices over the 15 yr of data. Figure 3 results from this calculation. Each point represents the annual average value of Z or M for the "year" starting with the month indicated on the time axis. [The curve for Z (35°–55°N) could be extended through March 1968 using additional daily 700-mb index data kindly made available by the Extended Forecast Division, National Weather Service.]

It is immediately obvious from figure 3 that the long-term behavior of Z is distinctly different from that of M. In Z, a quasi-periodic pattern exists with fluctuations of approximately 5–7 percent (0.4–0.6 m/s around an average of about 7.7 m/s). The most noticeable feature in the M curve is the large drop in value beginning in 1959 after a rather smooth and essentially constant level for the first 9 yr of record and the apparent recovery about 2 yr later leading to the highest values of record in the last year.

Even without calculating a correlation coefficient, it is apparent that these two indices have no obvious similarities on a time scale between 1 and 15 yr. This dissimilarity is not surprising, since we know of no good physical reason why such relationships should exist on that time scale. The relative intensity of the zonal and meridional exchange rate is apparently a result of the organization of the general circulation responding to the ever changing energy and momentum transport requirements at that time, and a 15-yr period is much too short to establish changes in this overall circulation character. One might hypothesize that long-term climatic changes may result in corresponding shifts of the zonal and meridional indices and thus lead to relationships that can be expressed in terms of correlation coefficients.

Since there are no common features in Z and M, it is practical to discuss the two time series independently. We have already mentioned the quasi-periodic character of the annual Z-averages. A simple count of the cycles (8+) in about 18 yr gives an average period of 23–24 mo (9 minima in 18 yr). However, if one considers the first three cycles (i.e., the November 1951, May 1953, August 1954 maxima) as something different (they are much less pronounced) and only takes into account the remaining cycles (5+), he gets a period of about 28 mo. Having had some experience with quasi-periodic phenomena, we believe that no unique true periodic behavior of this kind can be proven, despite a possible temptation to find here another case of a 26-mo periodicity. There is some indication that Z goes through cyclic variations that have an average time constant anywhere between 15 and 30 mo. This material is inadequate for more detailed resolution (if such a resolution has any physical meaning at all). The highest (8.25 m/s) and lowest (6.98 m/s) observed annual averages for Z are found in the last cycle (April 1965–March 1966 and March 1967–February 1968, respectively), indicating an increase in the amplitudes of these variations throughout the record.

The behavior of the annual averages of M is radically different. Little variation of consequence is noted in the early part of the record; some longer term trends could be suspected. In 1959, however, an abrupt decline of the M-values commences. A minimum M-value of approximately 6.3 m/s (a reduction of about 1 m/s or 15 percent) is reached late in the year. Low meridional indices are maintained throughout 1961, followed during the next 2 yr by a slow recovery. By early 1963, an M-level comparable to that prior to 1959 is reached. Further increases in M are noted throughout 1964 and the last value available (January 1965) is the highest (12 m/s) for the period of record.

This is a puzzling result, indeed, since there exists no evidence of extraordinary weather during the two crucial years, 1960–61. Furthermore, there is no indication in the behavior of Z that the zonal flow in these years was in any way unusual. We first suspected the data or the processing, but no errors could be found and we had to conclude that our data indeed contained this curious deficiency in M during the 1960–61 period. How unlikely the M-data are during this 2-yr period can be shown by comparing these 24 monthly averages with the corresponding ones for the previous 9 yr, 1951–59. Every one of the 24 values of M is below the appropriate 9-yr average, and by using the t-test one can show that the likelihood of these 24 values belonging to the same population as the 9 previous yr is far below 0.1 percent [$t=6.66$; $t(99.9)=3.77$].

Finally, the coincidence in the timing of this anomaly with the operational acceptance of objective analysis methods at the National Meteorological Center (NMC) leads us to recognize the reasons for our pattern. Recalling that M is proportional to $(\sum_{n=1}^{k} n^2 C_n^2)^{1/2}$, one can see that any new analysis technique, especially one that introduces computer methods with widespread objective interpolation and thus areal smoothing over regions with sparse data, will affect M. Such smoothing will suppress the higher order wave numbers which, in the expression for M, are seen to be weighted by the factor n itself. Since, on the other hand, Z is calculated from latitudinal average differences (wave number 0), no effect in Z can be expected. Thus, in these years, the meridional exchange appears to be clearly underestimated as a result of machine smoothing. Apparently, NMC realized this soon and modified their procedures, in a number of steps, so that they again reflected the same level of high wave number contributions as had been previously normal when these maps were analyzed by experienced human analysts. The final increase may or may not be spurious; we tend to regard it as an indication that (1) the objective interpolation schemes are now quite sensitive and that (2) more data are available, thereby further increasing the influence of the higher wave numbers.

This result, in a sense, is unfortunate since it eliminates, at least for discussion of the meridional index, a 2-yr time span from our not too extensive data bank. Worse, however, is the implication of this result with respect to studies of energy conversions between latitudes, levels, and wave numbers. Evidently, the break found here makes it rather difficult to compare such energetics studies from different time periods (e.g., pre-IGY and IGY data vs. results in the early 1960s). One must be aware of these problems which apparently are not restricted to the 700-mb level and take precautions to overcome them.

5. SHORT-TERM VARIATIONS IN *Z* AND *M* AND THEIR RELATIONSHIPS

Up to this point, the short-term variations in the two indices had been filtered out. Only the obvious and very high correlation between their annual variations was found. However, according to the earlier findings of Willett, one might expect a slight and possibly significant negative correlation in the day-to-day or 3-day to 3-day variation. To investigate this possibility, we used the difference of a 3-day average index from the 31-day average (centered on the middle day of the 3-day interval) as an index of the zonal or meridional character of this particular 3-day period. In this way, the long-term variation is effectively eliminated.

In an attempt to use only independent pairs of data for the computation of the correlation coefficients, we selected the values for the 5th, 15th, and 25th of each month. This effectively eliminates the persistence in the successive values of Z and M. We found that, on the average, the persistence (lag correlation) drops to nonsignificant levels at about the 6th day. Each season of each year is thus represented by nine value pairs and each year, by 36 value pairs.

One further subdivision of the data was deemed advisable. Since the actual value of the average monthly zonal

TABLE 1.—*Correlations between* M *and* Z *(by seasons, yr, and* Z*-classes). Values exceeding the 95-percent confidence limit are underlined.*

		1951	1952	1953	1954	1955	1956	1957	1958	1959	1962	1963	1964	1965	1966	Season
																[Wi (52)=XII 51, I+II 52]
Winter	A		−0.06		−0.49					−0.54					−0.05	
	B			+0.31		+0.38		−0.38			+0.40		+0.29			+0.115
	C						+0.02		<u>+0.72</u>			+0.62		+0.02		
Spring	A					−.47	−.47					−.58	−.05			
	B		−.23		−.37			+.56	−.45	−.07						−.077
	C	+0.30		−.38							+.28			+.29		
Summer	A						−.40		<u>−.81</u>				+.05	−.60		
	B		+.63	−.38	−.31						+.30	+.30				−.164
	C	−.17				+.25		+.03		−.58						
Fall	A			−.34								+.19	−.19	+.12		
	B		−.45		−.38		−.43		<u>−.86</u>	−.12						<u>−.238</u>
	C	−.09				−.47		−.51			+.18					
Yr (XII–XI)			−.08	−.09	<u>−.41</u>	−.12	−.25	−.01	−.16	<u>−.36</u>	+.25	+.32	+.02	+.02		All yr and all seasons, −0.061*.

*$\sigma(r)=\pm 0.046$ for $N=468$ pairs

index has been used to stratify data in many studies [e.g., when relating index values to other meteorological parameters (Wahl 1953*a*, 1953*b*)], we felt that the correlation between Z and M could also depend upon the actual above- or below-normal behavior of the zonal indices. Therefore, the average value of Z for each season was grouped according to 3 classes—A for all seasons with above-normal Z-values, B for all seasons with normal values, and C for seasons with below-normal zonal index averages. Eliminating the data from December 1959 through November 1961 left us with 13 yr; the highest 4 yr were called class A, the next 5 were class B, and the lowest 4 were class C. The results of these correlation computations are compiled in table 1. For example, the first correlation coefficient, −0.06 under winter 1952, refers to the correlation between the nine value pairs of Z and M for Dec. 5, 15, 25, 1951, Jan. 5, 15, 25, 1952, and Feb. 5, 15, 25, 1952. Since the average value of Z was above normal, it is listed under class A. The next one, winter of 1953, is in class B and has a value of +0.31.

The correlation coefficient for the 36 values of each year is given below the individual year (December 1950 was not available and thus a winter value for 1951 could not be obtained). Listed on the right-hand side of table 1 is an average correlation for each season (obtained from 117 pairs). Finally, the overall correlation (all years and all seasons together, $N=468$ pairs) is −0.061. This last result clearly shows that no significant correlation between Z and M exists, since $\sigma(r)=\pm 0.046$ (i.e., the correlation barely exceeds one sigma in value). However, even the other results appear to indicate no significance. We have underlined in table 1 all values that exceed the 95-percent confidence limit. Only three of the 52 correlations calculated for the individual seasons of the various years exceed this limit, and two of them are negative correlations. Only two of the 12 yr show significant values (above 95 percent) and only one of the four seasons could be considered significant, even at this low 95-percent level.

From the above results, one may conclude that the "usual" coincidence of high zonal and low meridional index is essentially nonexistent. Among the 52 seasonal correlations, there are 21 positive and 31 negative coefficients, The overall correlation of all data is slightly negative; but, for all practical purposes, the zonal and meridional short-term variations are uncorrelated. The only valid (and very highly significant) relationship between Z and M is their parallel annual variation, which is highest when the M-values lag the Z-values by approximately 24 days.

While this result is certainly of considerable interest, it still does not yet exhaust all possibilities. There may be some positive or negative correlation between Z and M when one lags Z versus M by a few days; the above results are for a lag of 0 days (i.e., simultaneous values). However, hand calculations performed on some of the data did not give any better results for lags up to ±5 days than the above-mentioned ones for lag 0. It was felt, therefore, that the extensive calculations necessary to definitely disprove this assumption were not warranted.

6. ZONAL AND MERIDIONAL INDICES, INDEPENDENT OF LATITUDE

The foregoing discussion was restricted to the conventional definition of the indices as used previously in numerous publications. Some of the problems associated with this specific definition have already been mentioned, especially the effect of the seasonal migration of the latitude of the maximum zonal flow. It, therefore, was deemed advisable to explore this facet in more detail by calculating the zonal flow [and, equally, the meridional (absolute) flow] for a number of latitude bands. The zonal and meridional indices for these latitude bands will be labeled u and v', respectively, replacing the Z and M used with the fixed latitude band, 35°–55°N. By modifying the constants in the formulas given earlier, we can obtain a mean zonal flow, u, for each 10° latitude band. Similarly, one can compute (as indicated before) a meridional flow value, v, for each latitude and obtain the average value, v', of this quantity for the same 10° band by averaging the three v-values for the latitudes, $\phi-5°$, ϕ, and $\phi+5°$. Figures 4 and

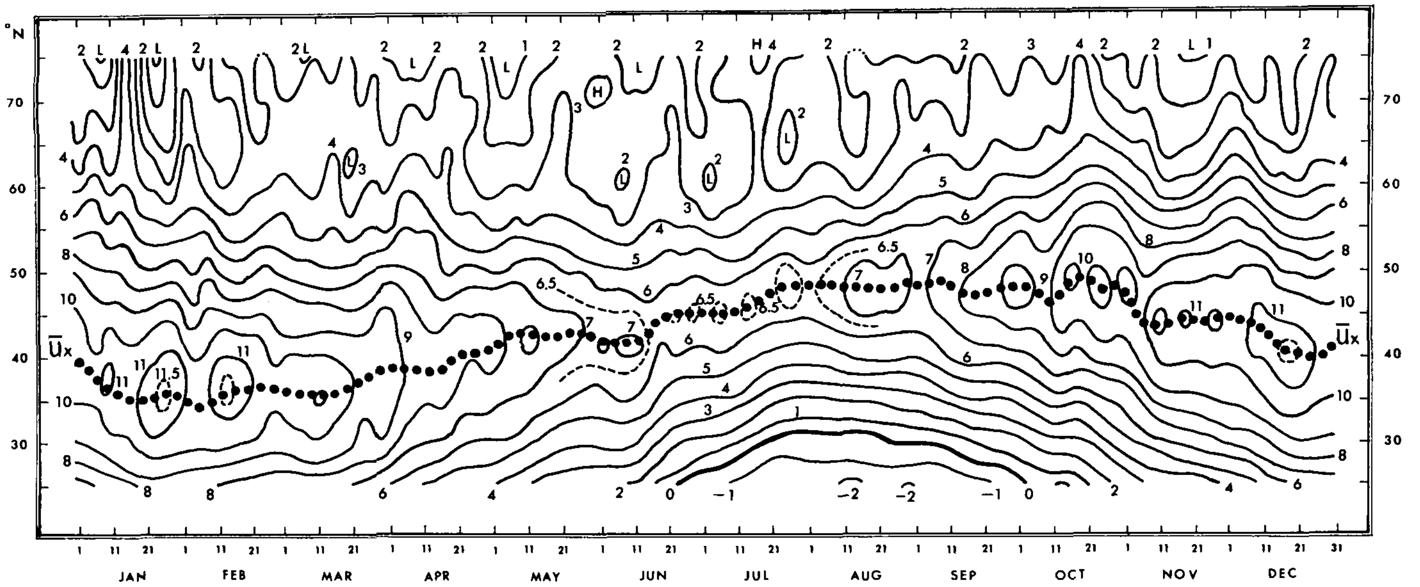

FIGURE 4.—Time vs. latitude isopleths (m/s) for 15-yr averages of $\bar{u}$ (hemispherically averaged geostrophic zonal wind at 700 mb) for 10° latitude bands.

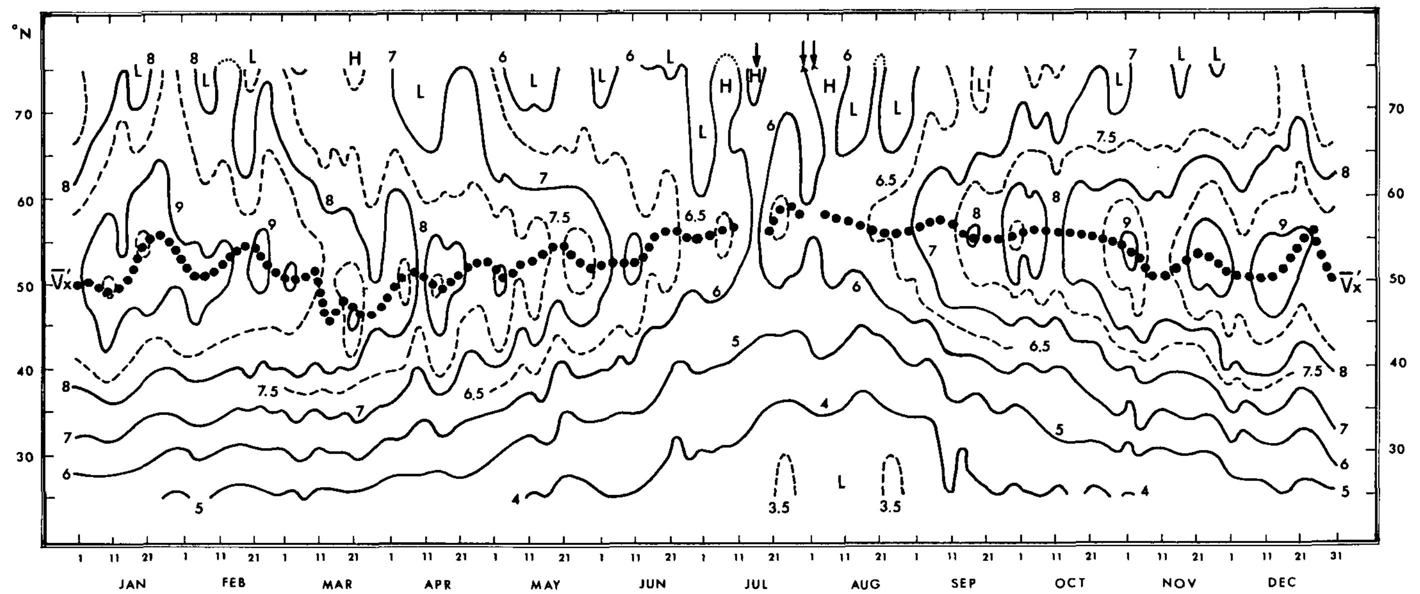

FIGURE 5.—Time vs. latitude isopleths (m/s) for 13-yr averages of $\bar{v}'$ [hemispherically averaged geostrophic meridional (root-mean-square) wind at 700 mb] for 10° latitude bands.

5 show the results of these calculations for the average year, based on the 1951–65 data. (The plot of the average year in v' is actually based on 13 yr, omitting 1960 and 1961 from the averages and other quantities discussed later.) Comparison of this 13-yr isopleth plot with one containing all 15 yr obtained earlier (not shown) showed the expected strong similarity down to small details such as individual centers of high or low values and the position of the maximum. The isopleth presentation of the results in figures 4 and 5 contains the latitudes of the interpolated maximum flow in addition to the actual (overlapping) 10° averages of zonal and meridional flow in meters per second. The interpolation was done by a parabolic fit to the three values surrounding the maxima. In some cases, the actual maximum values thus obtained also defined the exact delineation of the isolines.

A cursory glance at these two figures shows a number of interesting features. In figure 4, the change in the zonal flow maximum is by no means as smooth as one would expect in an average of 15 yr. There are several distinct times during winter when maximum zonal flow exceeds 11 m/s separated by intervals when the flow drops below this level. Values above 10 m/s appear around October 15, become widespread early in November, and then extend into the latter part of March. Between June 15 and August 10, the zonal average flow is below 7 m/s. From about June 20 through the end of September, the flow is easterly in the latitude band between 20° and 30°N. Between July 20 and August 20, the easterly flow extends northward into the 10° interval, 25°–35°N. The highest zonal westerly flow in these low latitudes (20°–30°N) occurs between January 20 and February 10.

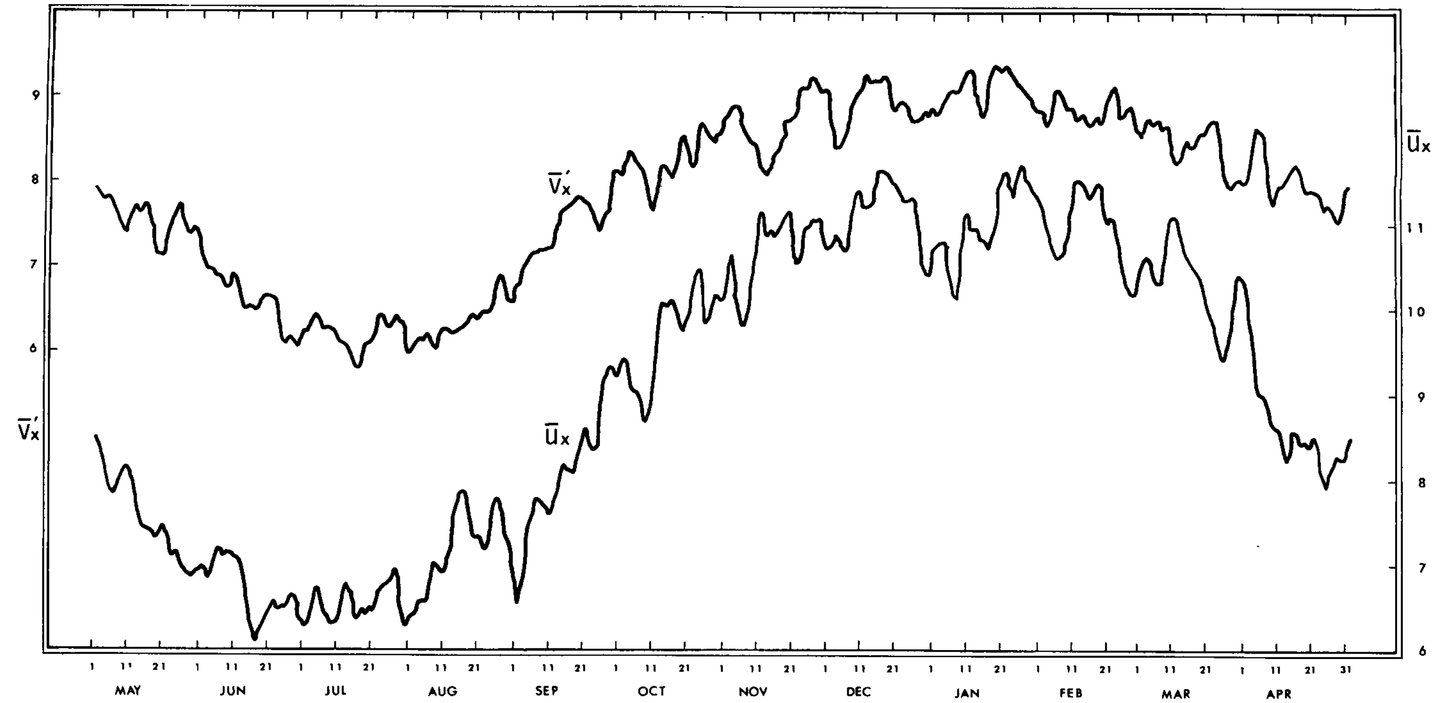

FIGURE 6.—Annual variation (year extends from May 1 to April 30) of $\overline{u}_x$ and $\overline{v}_x'$ (m/s).

The behavior of the zonal flow component north of the maximum is also of some interest. In the 10° band between 60° and 70°N, the variability of the mean flow with time is high, especially in December and January; values vary rapidly between more than 4 and less than 2 m/s. In fact, the overall pattern in the subpolar latitudes (north of 55°N) is confusing, with occasional high or low value intrusions from the north or south. The most remarkable of these variations is the strong increase in zonal flow at all latitudes north of 50°N around January 16, which may have some relationship to the January thaw singularity referred to in the literature (e.g., Wahl 1953*a*). At various times, a relative minimum of $\overline{u}$ appears around 60°–70°N (e.g., March 20, early June, July, August). In the middle of July, $\overline{u}$-values at 75°N are about 4 m/s, while they are lower by 1 to 2 m/s around latitude 60°–65°N. Apparently, this indicates the existence of a separate "arctic jet" in some of the years that make up this 15-yr average representation.

Of special interest is the latitudinal variation of the maximum zonal flow, $\overline{u}_x$. This quantity reaches its most southerly latitude of about 34°N in early February. It remains at its rather low winter latitude of 35°–36°N until mid-March, then progresses northward to about 42°N in early May, where it remains until early June when northward migration resumes. By the end of July, the typical summer latitude of about 47°N is reached. While, from about this time on, the value of $\overline{u}_x$ again begins to increase, the latitude of $\overline{u}_x$ remains at around 47°N until the end of October (with $\overline{u}_x$ having increased to more than 10 m/s). Then, the maximum migrates rapidly southward with a sharp retreat of nearly 4° in a week, remains stationary at 44°N until early December, and finally makes two more steplike southward migrations in the middle of December and the first 10 days in January to the 34°–35°N latitude typical for midwinter and late winter.

Thus, while the highest zonal flow occurs approximately when the flow maximum is at its most southerly location, the lowest zonal flow occurs long before the flow maximum reaches its most northerly position. In fact, the actual northernmost position is reached in mid-October, when the value of the zonal flow has already increased to nearly winter levels. This delay in the southward movement of $\overline{u}_x$ is probably related to the influence of the summer monsoon circulation over Asia which keeps the hemispherically averaged zonal flow maximum in this region still far north as long as southerly components in the geostrophic flow remain. It could also be related to the minimum of month-to-month persistence in climatic anomalies noted by Namias (1952) and Craddock and Ward (1962).

The characteristics of the meridional flow indices (fig. 5) are quite different from those of the zonal flow. The position of the maximum meridional flow value is not as well defined and its value varies with the seasons with a somewhat smaller amplitude. The highest values of $\overline{v}_x'$, somewhat above 9 m/s, first appear at the end of October; other maxima occur in late November and mid-December, and most of January has meridional flow values above 9 m/s. The maximum then declines, with a final peak occurring around the 20th of February [possibly related to the index cycle (Namias 1950) frequently observed at that time]. The lowest values during the year, slightly below 6 m/s, are found in mid-July. In winter, $\overline{v}'$-values in the 20°–30°N zone are around 5 m/s (mid-December to end of February), while in summer, they are typically between 3.5 and 4 m/s. North of the maximum, $\bar{v}'$-values are higher than in the south; they are also more variable from day to day. Occasionally, values as high as 8 m/s (i.e., only slightly lower than the maximum) occur in winter in latitudes as high as 70°–80°N. Even in summer, the difference between the maximum further south and the near-polar latitudes is often only a fraction of 1 m/s. In fact, on three occasions the highest $\bar{v}'$-value for the date was found at 75°N (indicated by the arrows on top of fig. 5).

The maximum meridional flow index is generally confined to the band between 50° and 60°N, with only rare excursions south of 50°N (early December and the second half of March). The most northerly position (58°N) of $\bar{v}_x'$ occurs in late July and early August, whereas the most southerly position (near 45°N) is found around March 25. One must conclude from these findings that not only is the usually used zonal index, Z, a poor indicator of the zonal flow character, as mentioned before, but that the meridional index, M, for the zone 35°–55°N also has shortcomings since this zone does not contain the maximum of this quantity for a considerable length of time (mainly in summer). If one wished to use a single index to describe the hemispheric flow character, it would appear that he should select the zone between 30° and 60°N.

To study the actual values of the maximum zonal ($\bar{u}_x$) and meridional ($\bar{v}_x'$) flow (10° latitude band), we have plotted the annual variation of these two quantities in detail as obtained from our calculations (fig. 6). The centered 3-day average value is represented for each day. For convenience, we plotted the average year starting with May 1, to preserve the continuity of the curves at the most interesting time, namely in midwinter and midsummer. Note that the scales of the two curves are offset to avoid possible overlap. Both curves show the high correlation of the annual variation already noted in the fixed latitude indices, the similarity of this annual pattern is even more pronounced in these two curves. Both reach a sort of plateau in November ($\bar{u}_x \approx 11$ m/s, $\bar{v}_x' \approx 9$ m/s) that is maintained until the end of February; both curves reach their lowest values (6.5 and 6.2 m/s, respectively) between the middle of June and early August.

One especially noteworthy feature, particularly in the $\bar{u}_x$ curve, is the apparent double maximum in winter, with the first high point being attained around December 15, followed by a distinct decline (by more than 1 m/s) to the beginning of January and a second high point around January 20–25. This pattern compares well with a similar pattern in the annual (smoothed) variation of the average zonal available potential energy (850–500 mb) derived from 5 yr of data by Krueger et al. (1965) which was also cited and compared to model calculation results by Kraus and Lorenz (1966). This pattern is apparently absent in the curve for $\bar{v}_x'$; however, in both curves one notices a rather pronounced quasi-periodic structure with occasional deep valleys and high peaks interrupting the smooth annual variations. More will be said about these "singularities" in a later paper.

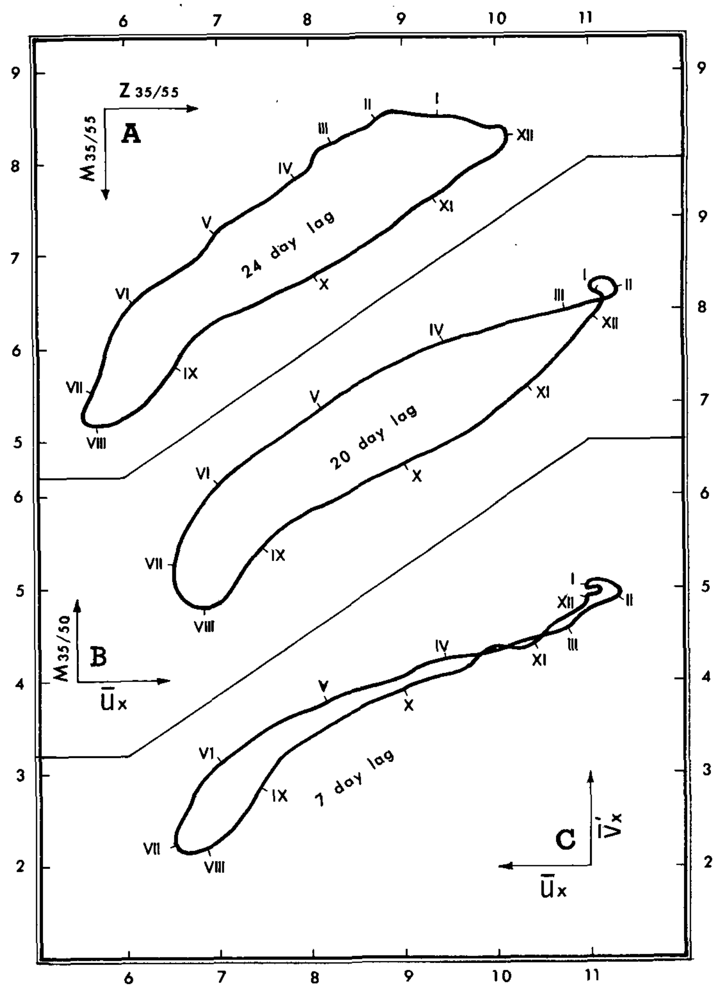

FIGURE 7.—Relationship between various zonal and meridional indices (m/s) for (A) Z(35°/55°) vs. M(35°/55°), (B) $\bar{u}_x$ vs. M'(35°/50°), and (C) $\bar{u}_x$ vs. $\bar{v}_x'$.

7. CORRELATION BETWEEN THE LATITUDE-VARIABLE INDICES

When discussing the standard indices and their annual variations, we showed that both quantities follow a similar annual pattern with a maximum in winter and a minimum in summer so that the correlation between the two indices is highly positive. We also mentioned that the best correlation was obtained when M lagged Z by approximately 25 days. This time lag relationship also can be shown graphically by plotting, in a Z-M coordinate system, the simultaneous values of the two quantities for the course of the average year as shown in figure 7A. The lag relationship is quite obvious in the hysteresis-like shape of this curve; it also can be seen that this lag is present throughout the full year. In a similar manner, one can also correlate the latitude-independent quantity $\bar{u}_x$ with a meridional index quantity, the question being only which particular meridional index one should use to relate to the maximum zonal flow index. Two such correlations are shown in figures 7B and 7C. Figure 7B presents the relationship of $\bar{u}_x$ to the meridional index for the latitude band 35°–50°N (M' 35/50); that is, for

TABLE 2.—*Lag correlation coefficients. Maximum coefficients for each case are underlined.*

Lag (days)	A	B	C	Case	
0	0.867	0.928	0.966	A	$Z(35°/55°)$ vs. $M(35°/55°)$
5	.903	.956	.972		
10	.936	.973	.971	B	$\bar{u}_x$ vs. $M'(35°/50°)$
15	.954	.984	.964		
20	.964	.988	.946	C	$\bar{u}_x$ vs. $\bar{v}'_x$
25	.967	.984	—		
30	.962	.973	—		

the latitude zone in which $\bar{u}_x$ is usually located throughout the year. Figure 7C shows the relationship between $\bar{u}_x$ and $\bar{v}'_x$. The computational results of lag correlation of these three cases are given in table 2.

The highest correlation in case (A) occurs at a lag of about 24 days; in case (B) the lag is 20 days, while in case (C) the best correlation already occurs at approximately 7 days. This, however, is the "average lag" over the whole year; in figures 7A–7C, one can see that, in both case (B) and case (C), the lag is smaller in winter and larger in summer. In the case of correlating $\bar{u}_x$ and $\bar{v}'_x$, there is no lag from about October 15 to the end of March, while a remnant of the much more pronounced lag relationship in summer still exists even in this combination, thus leading to the average lag of 7 days. One might ask, however, what the real physical meaning of this latter combination is, since this correlation is between values of $\bar{u}_x$ and $\bar{v}'_x$ at different and usually well-separated latitudes. The most meaningful relationship is probably case (B) where the maximum zonal flow, $\bar{u}_x$, is correlated to the meridional index, M', of the same latitudinal band in which this maximum is reached. The significance of this relationship is also indicated by the fact that, at the best lag, the correlation is highest (+0.988). Note that the correlations were obtained not from the individual 3-day running averages over the 15 yr that are plotted in figure 6 but from heavily smoothed values (by calculating 35-day running averages and selecting 73 values; i.e., every 5th day). The resulting very high correlation coefficients, in essence, therefore, represent only the annual cycles. However, even if one correlates the two curves on figure 6 (by taking the average value of $\bar{u}_x$ and $\bar{v}'_x$ every third day), the correlations are nearly as high. For example, for the smooth curves, we find $r(\bar{u}_x/\bar{v}'_x)=0.966$ for lag 0; using the 122 unsmoothed 3-day averages, $r=0.936$.

8. STANDARD DEVIATION OF u AND v' BY LATITUDE AND SEASON

It was already mentioned that the variations in the values of $\bar{u}$, especially from day to day, are large and apparently are difficult to account for by simple random fluctuations in the 15-yr record. To establish this in a more general way, we have calculated and plotted the two isopleths for the standard deviation of u and v' (figs. 8, 9). The standard deviation for each x (i.e., u or v') is defined as

$$\sigma(x)=\left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{1/2}$$

where the x_i represents the values of the variable at the same 3 consecutive days in each of the 15 (or 13) yr, $\bar{x}$ is their average, and n equals 3 times the number of years used (i.e., 45 or 39). The sequence of dots in figure 8 indicates the latitude of $\bar{u}_x$.

The standard deviation $\sigma(u)$ again shows the expected annual and latitudinal variation. Generally speaking, $\sigma(u)$ is large in near-polar latitudes and smaller in more southerly latitudes; winter values of $\sigma(u)$ are somewhat higher than summer values. This general pattern, however, is not the only variation that can be seen; the occasional rather drastic changes in $\sigma(u)$ within a matter of a few days are much more evident than the slowly changing annual variation. They apparently indicate times of increased tendency for "shifting gears" in the general circulation.

One such break occurs at the end of February and into early March. A separate maximum of $\sigma(u)$ appears south of the latitude of $\bar{u}_x$, accompanied by another maximum at about 55°N. These two maxima are clearly related and indicate the time at which the maximum zonal flow begins shifting from the southerly to a more northerly position. In some years, this shift occurs earlier and in some, a few days later. Again, one sees here the well-known effect of the index cycle first mentioned by Namias (1950). Slightly later in March, the highest $\sigma(u)$ values (>4 m/s) are found in subpolar latitudes; this occurs exactly at the time when the polar night has ended and rapidly increasing daylight hours begin to change the energy budget of these latitudes.

The next break occurs between the middle of April and the first half of May. First, the value of $\sigma(u)$ in the middle latitudes decreases to a relative minimum of about ± 2 m/s at the end of April and then again increases to a relative maximum of ±2.5 m/s (at 55°–60°N). At the same time, another maximum appears south of the latitude of $\bar{u}_x$. Shortly thereafter, the typical low summer values are established, with $\sigma(u)$ rarely approaching or surpassing ± 2 m/s south of 65°N. The lowest $\sigma(u)$-values in the summer occur south of 35°N (less than ± 1 m/s during July and August). Here, $\bar{u}$ is either weak westerly or easterly (i.e., negative). At the end of September, $\sigma(u)$ undergoes a rapid increase to more than ± 2 m/s at latitudes 55° and 60°N (i.e., about to the usual wintertime level). Other increases throughout the fall and early winter occur at the end of November somewhat further south and around December 15 when $\sigma(u)$ at 75°N first reaches ± 4 m/s and at 60° and 55°N exceeds ± 2.5 m/s for the first time in winter. During this winter period, $\sigma(u)$ at the latitude of the u-maximum reaches its highest value for the year, exceeding ± 2 m/s.

The comparable quantity for v', the standard deviation $\sigma(v')$ in figure 9, shows very little if any relationship to either $\bar{v}'$ or $\bar{u}$ or to $\sigma(u)$. When analyzing the $\sigma(v')$ isopleths, we found that the calculated values had a

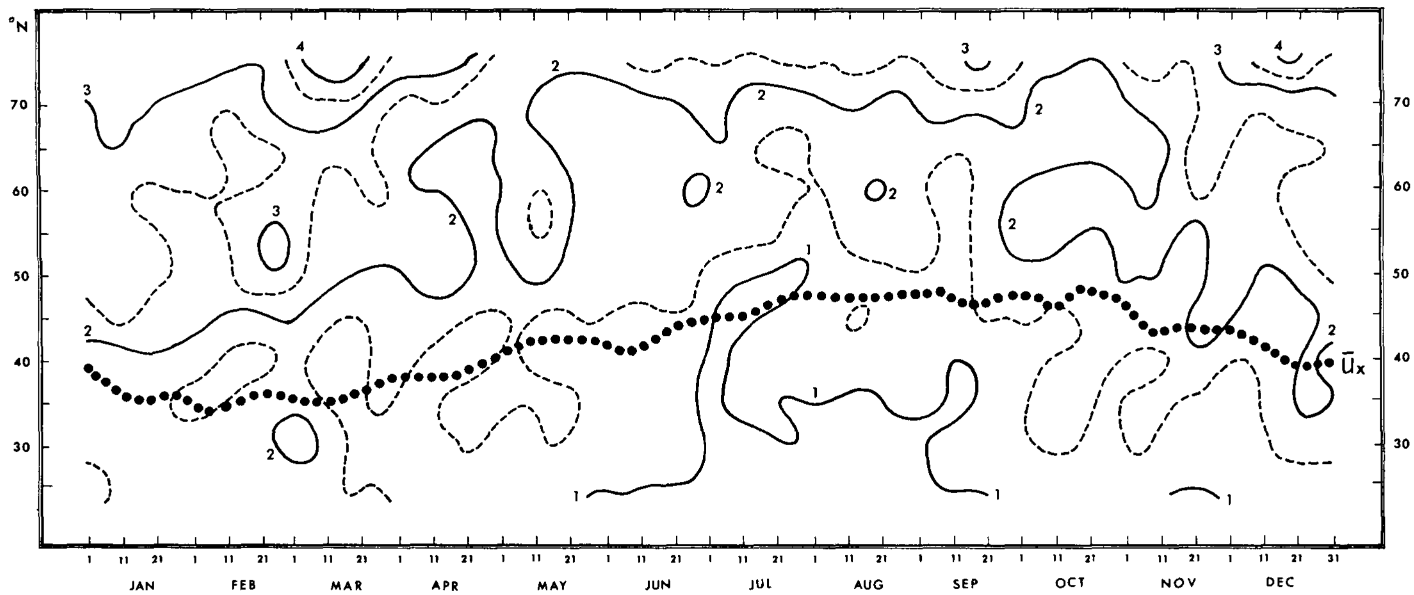

FIGURE 8.—Time vs. latitude isopleths (m/s) of standard deviation, $\sigma(u)$. Location of $\bar{u}_x$ (dotted curve) is from figure 4.

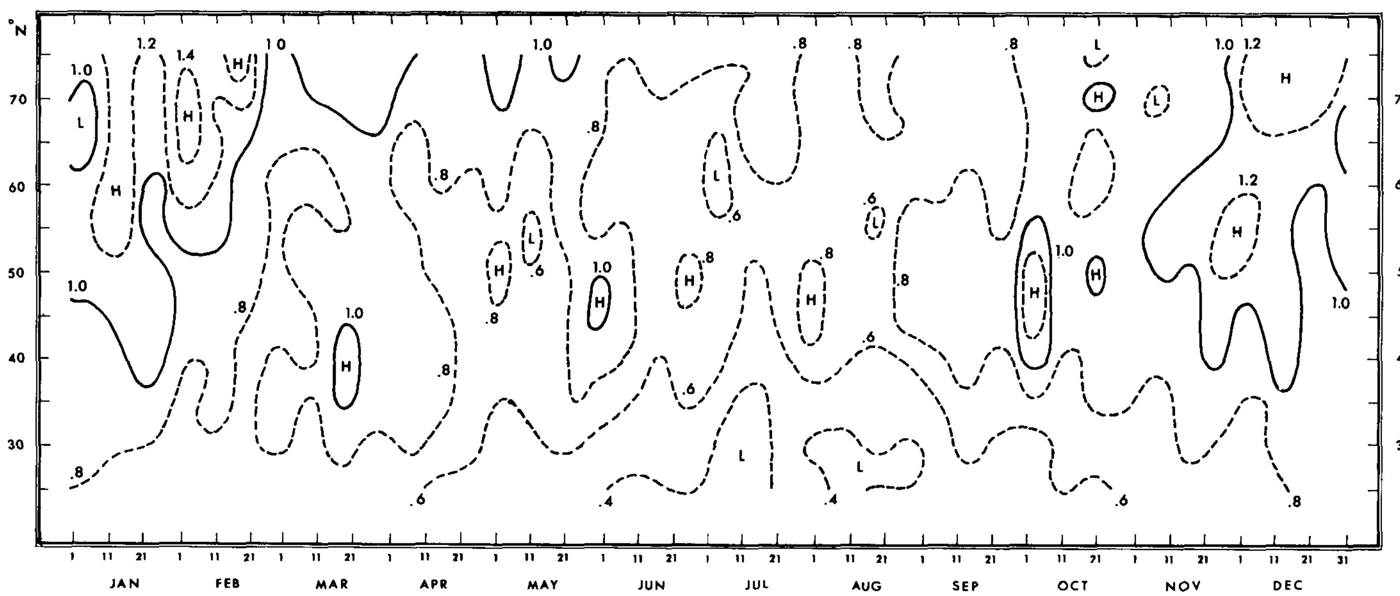

FIGURE 9.—Time vs. latitude isopleths (m/s) of standard deviation, $\sigma(v')$.

complicated and variable day-to-day pattern. This variability is undoubtedly due primarily to the effect of the high wave number amplitudes on $\bar{v}'$; some of this variability is also a result of noise in the basic data. To bring out the large-scale pattern in $\sigma(v')$, we applied rather drastic smoothing techniques, calculating the average of three independent, consecutive 3-day average values of $\sigma(v')$ at each latitude. Unfortunately, this also decreases the values of $\sigma(v')$, especially in the extreme value areas. However, the overall patterns should remain the same with only minor shifts in the date of occurrences for such extreme values.

In general, the largest values of $\sigma(v')$ occur in the subpolar latitudes in winter. A secondary $\sigma(v')$-maximum (which occasionally takes over as the prime maximum) occurs during much of the year in middle latitudes between 35° and 55°N. On the basis of these smoothed values, $\sigma(v')$ ranges from about $\pm$ 1.4 m/s at 70°N in early February to 0.3 m/s at 25°N in midsummer. The time history of $\sigma(v')$, like that of $\sigma(u)$, displays occasional strong "breaks" in the pattern at specific times, indicating the above-mentioned shift in the circulation. In mid-July, both $\bar{v}'$ and $\sigma(v')$ are at a minimum at nearly all latitudes, indicating a minimum value of meridional transport and a small interannual variability in this transport. Physically, this relates to the minimum of the south-north mid-tropospheric temperature and pressure gradients observed during this season.

From about mid-July on, a slow but rather steady increase of $\sigma(v')$ is noticeable; that is, from 0.6 to 0.9 m/s at 50°N. An impressive sudden surge of $\sigma(v')$ is encountered in the first days of October. In a 9-day period, $\sigma(v')$ increases by more than 50 percent (from 0.89 to 1.32 m/s) in the center of a distinct maximum between 45° and 50°N, and the $\sigma(v')$-value in all but the most southerly latitudes exceeds 0.8 m/s. This sudden increase can only be termed extreme in nature; values of this magnitude in $\sigma(v')$ do not recur at these latitudes at all, even during the height of the winter. This extreme surge occurs at a time when $\bar{v}'$ itself is rapidly increasing and attaining a first significant maximum well above 8 m/s. This extreme peak of $\sigma(v')$ subsides to "normal" levels shortly after October 8–10; values slowly increase again during the rest of October and into early November. Then, a new rapid increase sets in and by the end of November the typical winter values above $\pm$ 1 m/s are reached in all latitudes north of about 45° or 50°N. This coincides with the ocurrence of the first maximum above 9 m/s in $\bar{v}'$.

When $\bar{v}'_x$ itself drops, during late December and early January, so does $\sigma(v')$; the next increase in $\bar{v}'$ is also accompanied by an increase in $\sigma(v')$. After January 15, a decline of $\sigma(v')$ in all latitudes begins; by the end of the month, values are below $\pm$ 1 m/s in all latitudes south of 55°N. The next increase [to the absolute highest $\sigma(v')$ values in the year—larger than 1.5 m/s at 65°N] encompasses chiefly the subpolar latitudes, while the values south of 50°N continue to drop to less than 0.8 m/s by the end of February, by which time the last surge of $\sigma(v')$ in polar latitudes has also subsided. The next strong change, similar to the surge in October, occurs in low latitudes (35°–40°N) around March 20. Again, a separate maximum forms with values well above 1 m/s while values before were as low as 0.75 m/s. By the middle of April, values of $\sigma(v')$ at all latitudes south of 60°N have dropped to below 0.8 m/s (i.e., typical summer values), with the exception of an isolated 1.0-maximum appearing near 45°N at the end of May. This latter increase may be related to the northward surge of the monsoon circulation of the Eurasian continent and the comparable feature in the Southwest of North America.

The October singularity in $\sigma(v')$, and to a certain extent in $\bar{v}'$ itself, can be studied in detail by looking at the data for the individual 13 yr used to calculate σ. In every one of these years, a significant change of v' (averaging 3.6 m/s at 50°N) occurred sometime between September 25 and October 10. In 9 of the 13 yr, a change was in progress (either up or down) between September 30 and October 3. Thus, strong intensification of $\sigma(v')$ during these days is the result of a small variation in the date of this first significant change in v' in the fall which is reflected in the $\bar{v}'$-isopleths by the rapid increase of $\bar{v}'$ to an early maximum value. Synoptically, one might equate this occurrence of a $\bar{v}'$-maximum with a brief intense burst of increased north–south exchange in the form of intensified synoptic scale circulation features that subside rather rapidly afterwards. This can also be compared with the "equinoctial storminess" heralding the very first wave of winter-strength circulation modes which is often followed by a more quiet period of "Indian Summer" later in October. The counterpart of this "beginning of winter" is the March singularity indicating the last widespread winter mode occurrence, again at about the time of the equinox. Even the latitudinal arrangement is correct; the winter begins in the north around 50°N, whereas the end of winter is first found in the lower latitudes of 35°–40°N.

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